



Multi-Player Bandits Revisited

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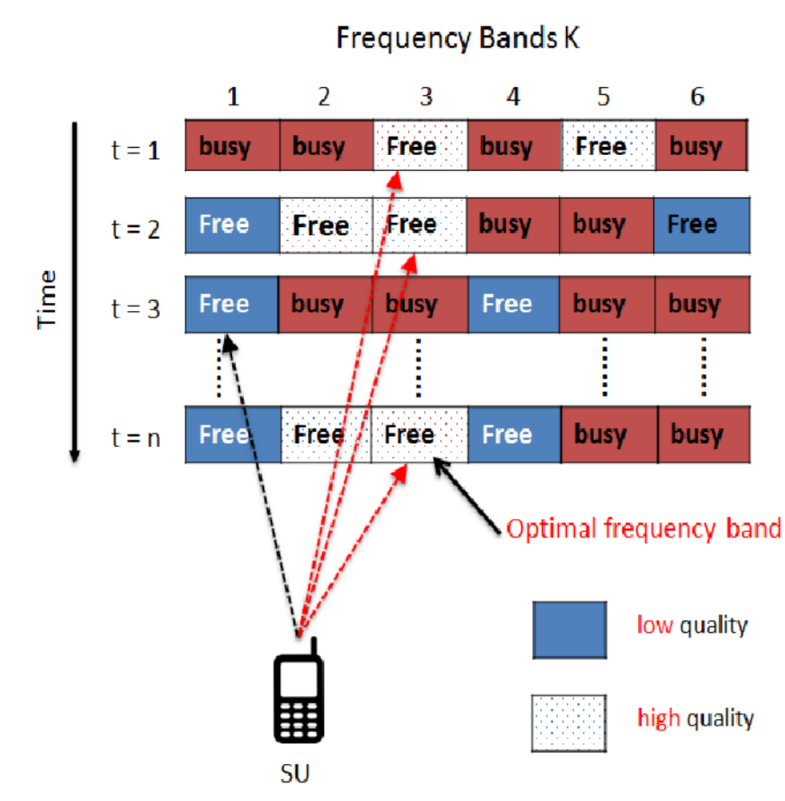
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1. INTRODUCTION & GOAL

Goal: insert objects in a wireless network, keep a good *Quality of Service*.





- *Hypothesis:* object j choose channel $A_j(t) \in \{1, \dots, K\}$, to use to communicate at time t .
- *Idea:* use on-line **Machine Learning algorithms** ?
- *Not so easy:* each device takes its own decisions, without central control or communication, has light CPU/memory etc.
- \Rightarrow **Solution: Decentralized MAB algorithms !**

2. MODEL: TIME/FREQUENCY PROTOCOL



Fix-duration communication. Channels can be free or busy.

M PLAYERS IN THE NETWORK

- K RF channels (of same bandwidth), e.g., $K = 9$.
- Primary users  create a background traffic.
- Channel k is busy with mean $\mu_1, \dots, \mu_K \in [0, 1]$.
- Sensing gives binary feedback $Y_{k,t} \sim \mathcal{B}(\mu_k)$.
- $1 \leq M \leq K$ secondary users , $j \in \{1, \dots, M\}$.
- Base station  replies if  packet is received.
- Collision indic. $C^j(t) = \mathbb{1}(\exists j' \neq j, A^j(t) = A^{j'}(t))$.
- Non-stochastic 0/1 reward $r^j(t) := Y_{A^j(t),t} \times C^j(t)$.

3. PREVIOUS WORKS (SINCE 2009)

Ideas: 1) use empirical means to learn the M best arms (μ_1^*, \dots, μ_M^*), 2) and each user orthogonalize on this set.

- RhoRand: use an index policy (UCB) and random *ranks*, [Anandkumar et al, 2011]
- MEGA: use a simple ε -greedy algorithm (hard to tune, and not so efficient), [Avner & Mannor, 2015]
- Musical Chair: pure exploration then random hopping until convergence on M best arms, [Shamir et al, 2016]
- Some others: TDFS uses a time sharing, RhoLearn uses BayesUCB to learn the ranks, etc.

4. OUR PROPOSAL

- 1) kl-UCB $>$ UCB₁ for best arms identification.
- 2) New algorithm MCTopM for orthogonalization:
 - Estimate the set of M best arm $\widehat{M}^j(t)$,
 - Randomly play in $\widehat{M}^j(t)$ until fixed on a good one (no collision: fixed on a “chair”).

5. UCB₁ AND kl-UCB INDEXES

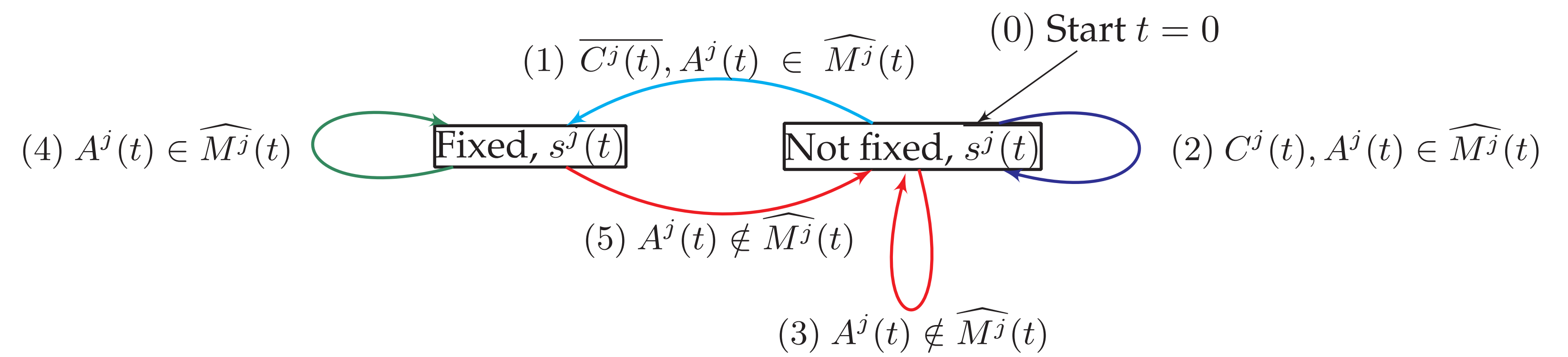
Let $T_k^j(t)$ the selections of channel k for player j , $X_k^j(t)$ mean of free sensing.

$$\text{UCB}_1 : g_k^j(t) := \frac{X_k^j(t)}{T_k^j(t)} + \sqrt{\frac{\log(t)}{2T_k^j(t)}}$$

$$\text{kl-UCB} : g_k^j(t) := \sup_{q \in [0,1]} \left\{ q : \text{kl} \left(\frac{X_k^j(t)}{T_k^j(t)}, q \right) \leq \frac{\log(t)}{T_k^j(t)} \right\}$$

[Garivier & Cappé, 2011], [Cappé et al, 2013]

7. ILLUSTRATION OF MCTOPM



6. MCTOPM ALGORITHM

```

1  $A^j(1) \sim \mathcal{U}(\{1, \dots, K\})$ ,  $C^j(1) = s^j(1) = \text{False}$ 
2 for  $t = 0, \dots, T - 1$  do
3   if  $A^j(t) \notin \widehat{M}^j(t)$  then // (3) or (5)
4      $A^j(t+1) \sim \mathcal{U}(\widehat{M}^j(t) \cap \{k : g_k^j(t-1) \leq g_{A^j(t)}^j(t-1)\})$ 
5      $s^j(t+1) = \text{False}$  // arm with smaller UCB at  $t-1$ 
6   else if  $C^j(t)$  and  $\overline{s^j(t)}$  then // (2)
7      $A^j(t+1) \sim \mathcal{U}(\widehat{M}^j(t))$ 
8      $s^j(t+1) = \text{False}$ 
9   else // transition (1) or (4)
10     $A^j(t+1) = A^j(t)$  // same arm
11     $s^j(t+1) = \text{True}$  // on “chair”
12  end
13  Play arm  $A^j(t+1)$ , get new observations,
14  Compute the indexes  $g_k^j(t+1)$ , and set  $\widehat{M}^j(t+1)$  for next step.
15 end

```

8. THEOREMS

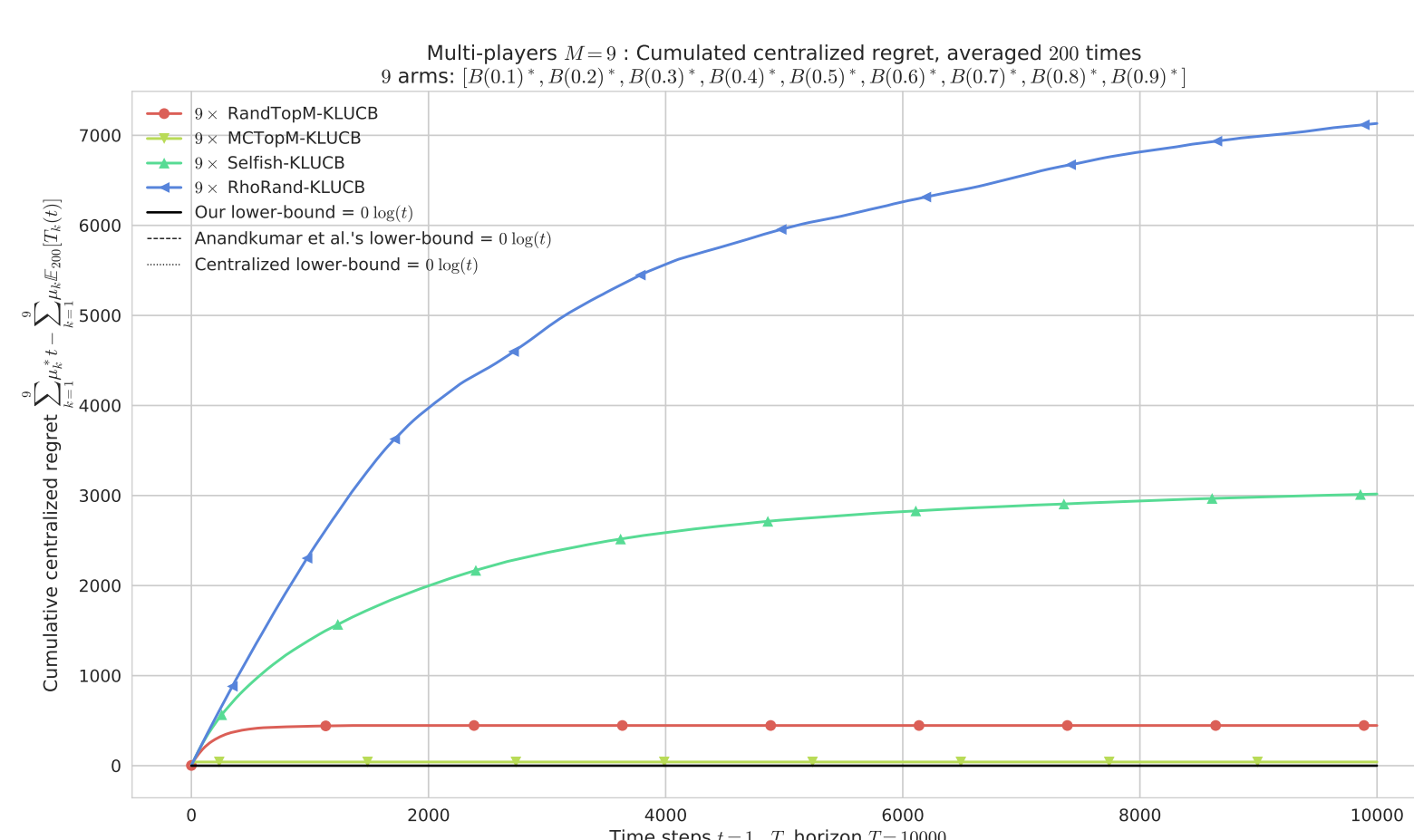
Let $T_k(T) := \sum_{j=1}^M T_k^j(T)$ total selections of arm k , and $\mathcal{C}_k(T)$ counts collisions on arm k .

1. Regret: $R_T(\mu, M) := \mathbb{E}_\mu \left[\sum_{t=1}^T \sum_{j=1}^M \mu_j^* - r^j(t) \right] = \left(\sum_{k=1}^M \mu_k^* \right) T - \mathbb{E}_\mu \left[\sum_{t=1}^T \sum_{j=1}^M r^j(t) \right]$

$$R_T = \sum_{k \in M\text{-worst}} (\mu_M^* - \mu_k) \mathbb{E}_\mu[T_k(T)] + \sum_{k \in M\text{-best}} (\mu_k - \mu_M^*) (T - \mathbb{E}_\mu[T_k(T)]) + \sum_{k=1}^K \mu_k \mathbb{E}_\mu[\mathcal{C}_k(T)].$$
2. Lemma for upper-bound: only two terms to focus on (bad selections and collisions).

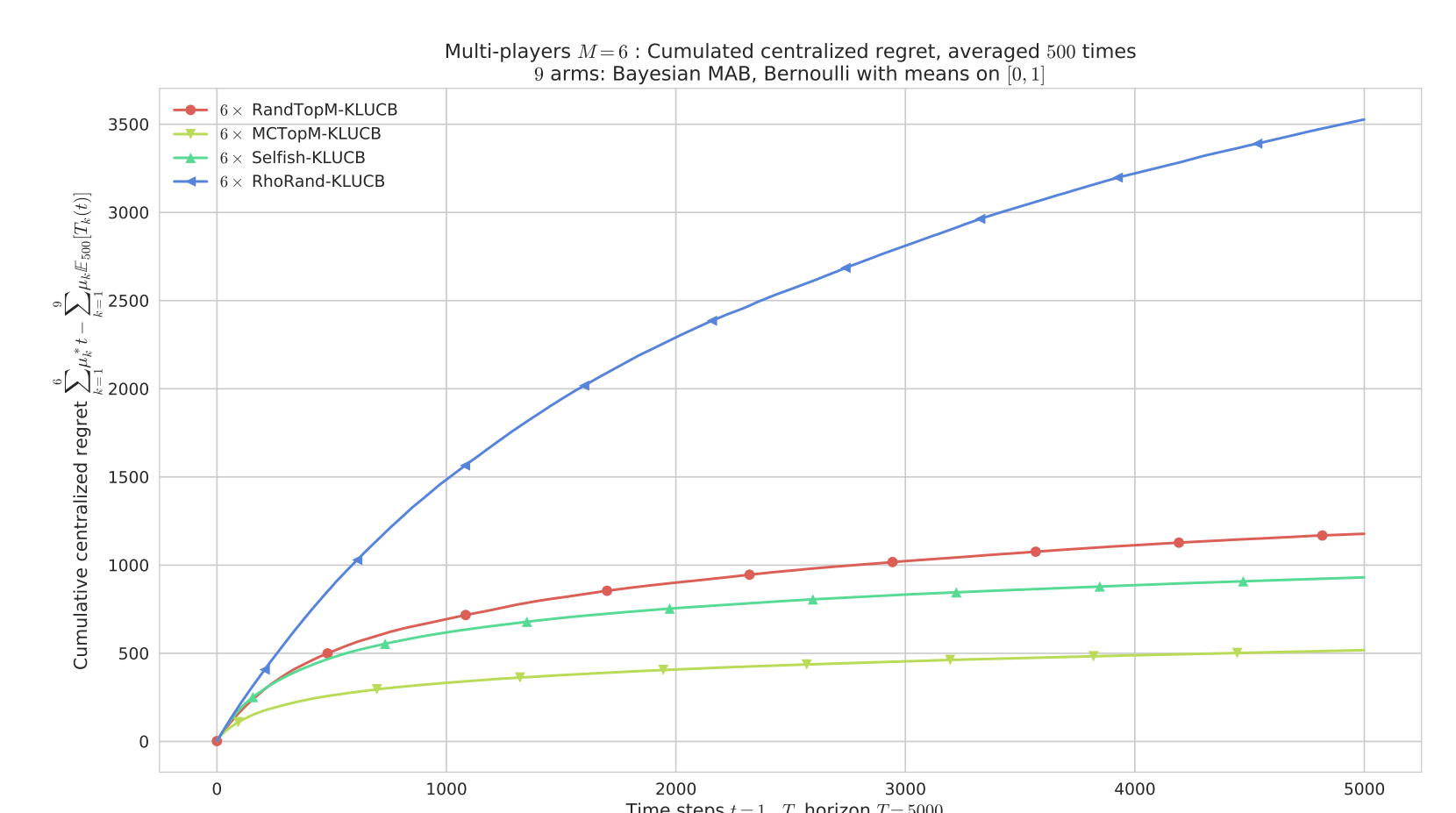
$$\sum_{k \in M\text{-best}} (\mu_k - \mu_M^*) (T - \mathbb{E}_\mu[T_k(T)]) \leq (\mu_1^* - \mu_M^*) \left(\sum_{k \in M\text{-worst}} \mathbb{E}_\mu[T_k(T)] + \sum_{k \in M\text{-best}} \mathbb{E}_\mu[\mathcal{C}_k(T)] \right).$$
3. If all M players use MCTopM with kl-UCB, $\forall \mu, \exists G_{M,\mu}, R_T(\mu, M) \leq G_{M,\mu} \times \log(T) + o(\log T)$.

9.1. ILLUSTRATION FOR $M = K$



Only RandTopM and MCTopM achieve constant regret in this saturated case (proven).

9.2. ILLUSTRATION FOR $M < K$



RhoRand < RandTopM < Selfish < MCTopM.

10. CONCLUSIONS

- Our algorithm MCTopM is uniformly better than all previous proposals.
- *Good bounds:* $\log(T)$ collisions, arm switches, bad selections and regret.
- Real-world implementation? \leftrightarrow Yes, presented at ICT 2018!
- *Future?* learn M , arrival/departures of users, dynamic problems, jammers etc.

11. MAIN REFERENCES

MORE ON-LINE \rightarrow <http://lbo.k.vu/JdD2018>

- [BBM⁺17] R. Bonnefoi, L. Besson, C. Moy, E. Kaufmann, and J. Palicot (2017). *Multi-Armed Bandit Learning in IoT Networks: Learning helps even in non-stationary settings*. In *12th EAI Conference on Cognitive Radio Oriented Wireless Network and Communication*.
- [BK18] L. Besson and E. Kaufmann (April 2018). *Multi-Player Bandits Revisited*. In *Algorithmic Learning Theory*. Lanzarote, Spain. URL <https://hal.inria.fr/hal-01629733>.
- [B18] Simulation code on [GitHub.com/SMPyBandits/SMPyBandits](https://github.com/SMPyBandits/SMPyBandits), open source (MIT license)!

12. THANKS TO ...

- Organizers of the *Workshop on MAB and Learning Algorithms!*
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